



Problem of the Week

Problem C and Solution

Just Your Average Sequence

Problem

In a sequence of 6 numbers, every number after the first two is the average of the previous two numbers. The 4th number in the sequence is 22 and the 6th number in the sequence is 45. Determine all 6 numbers in the sequence.

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Solution

If x is the average of two numbers y and z , then $\frac{y+z}{2} = x$.

It follows that $y+z = 2 \times x$.

This idea is used in both solutions.

Solution 1

In the first solution, we solve the problem by working backwards.

Since the 6th number in the sequence is equal to the average of the two previous numbers, the 6th number must be the average of the 4th and 5th numbers.

So the sum of the 4th and 5th numbers must be 2 times the 6th number, or $2 \times 45 = 90$.

Therefore, the 5th number is $90 - 22 = 68$.

We now determine the 3rd number. The 5th number in the sequence is the average of the 3rd and 4th numbers. So the sum of the 3rd and 4th numbers is 2 times the 5th number, or $2 \times 68 = 136$. Therefore, the 3rd number is $136 - 22 = 114$.

We now determine the 2nd number. The 4th number in the sequence is the average of the 2nd and 3rd numbers. So the sum of the 2nd and 3rd numbers is 2 times the 4th number, or $2 \times 22 = 44$. Therefore, the 2nd number is $44 - 114 = -70$.

We now determine the 1st number. The 3rd number in the sequence is the average of the 1st and 2nd numbers. So the sum of the 1st and 2nd numbers is 2 times the 3rd number, or $2 \times 114 = 228$. Therefore, the 1st number is $228 - (-70) = 228 + 70 = 298$.

The sequence of 6 numbers is 298, -70, 114, 22, 68, 45.

We can indeed check that in this sequence each number after the first two is equal to the average of the previous two numbers.





Solution 2

We will now present a similar, but more algebraic solution.

Let the sequence be

a	b	c	22	d	45
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where a represents the 1st number, b represents the 2nd number, c represents the 3rd number and d represents the 5th number in the sequence.

We again solve this problem by working backwards.

Since the 6th number in the sequence is equal to the average of the 4th and 5th numbers, we have $45 = \frac{22+d}{2}$. Multiplying both sides by 2, we obtain $22 + d = 45 \times 2 = 90$. Rearranging, $d = 90 - 22 = 68$.

Therefore, the 5th number in the sequence is 68.

We now determine the 3rd number. Since the 5th number in the sequence is equal to the average of the 3rd and 4th numbers, we have $68 = \frac{c+22}{2}$. Multiplying both sides by 2, we obtain $c + 22 = 68 \times 2 = 136$. Rearranging, $c = 136 - 22 = 114$.

Therefore, the 3rd number in the sequence is 114.

We now determine the 2nd number. Since the 4th number in the sequence is equal to the average of the 2nd and 3rd numbers, we have $22 = \frac{b+114}{2}$. Multiplying both sides by 2, we obtain $b + 114 = 22 \times 2 = 44$. Rearranging, $b = 44 - 114 = -70$.

Therefore, the 2nd number in the sequence is -70 .

We now determine the 1st number. Since the 3rd number in the sequence is equal to the average of the 1st and 2nd numbers, we have $114 = \frac{a + (-70)}{2}$. Multiplying both sides by 2, we obtain $a + (-70) = 114 \times 2 = 228$. Rearranging, $a = 228 + 70 = 298$.

Therefore, the 1st number in the sequence is 298.

Therefore, the sequence of 6 numbers is 298, -70 , 114, 22, 68, 45.

We can indeed check that in this sequence each number after the first two is equal to the average of the previous two numbers.

298	-70	114	22	68	45
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