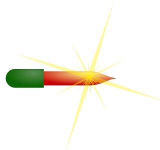


Problem of the Week
Problem A and Solution
Holiday Lights

Problem

Hunter needs exactly 6 metres of twinkle lights to decorate the roof-line of his house for the holidays. He has 3 strings of lights. The first string of lights covers 250 cm.

Which of the following options could represent the lengths of the other two strings of lights? Explain your thinking.

a) 175 cm and 175 cm

b) 150 cm and 150 cm

c) 180 cm and 170 cm

d) 150 cm and 200 cm

e) More than two of the above options are possibilities.

f) None of the options above are possibilities.

Solution

We know that 1 m = 100 cm, so 6 m = $6 \times 100 = 600$ cm. Since Hunter already has a 250 cm string of lights, he needs the other two strings to cover the remaining $600 - 250 = 350$ cm of his roof-line. We can check the four pairs of lengths to see how many of them total 350 cm.

a) $175 + 175 = 350$ cm
b) $150 + 150 = 300$ cm
c) $180 + 170 = 350$ cm
d) $150 + 200 = 350$ cm

From these calculations, we see that the total for three of these choices give us the required 350 cm. So the correct answer to the original question is e), since three of the first four options would work.

Alternatively, we could add 250 cm to each of the pairs of lengths listed in the problem to see which triples would give us a total 600 cm.

a) $175 + 175 + 250 = 600$ cm
b) $150 + 150 + 250 = 550$ cm
c) $180 + 170 + 250 = 600$ cm
d) $150 + 200 + 250 = 600$ cm

We see that three of the four possibilities provide the required 6 m. So the correct answer to the original question is e), since three of the first four options would work.

Teacher's Notes

We can break up numbers into smaller units in a variety of ways. A *factorization* of an integer is a combination of positive integers (*factors*) that can be multiplied together to produce the original value. For example, we can factor 6 in the following ways:

$$1 \times 6$$

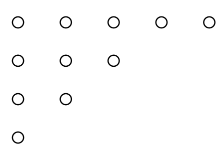
$$2 \times 3$$

The Holiday Lights problem essentially asks students to identify *partitions* of 600. A partition is a combination of positive integers (*parts*) that can be added together to produce the original value. For example, we can partition 6 in the following ways:

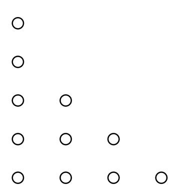
$1+1+1+1+1+1$	$1+1+4$	$1+2+3$	$3+3$
$1+1+1+1+2$	$1+5$	$2+2+2$	6
$1+1+1+3$	$1+1+2+2$	$2+4$	

Multiplication and addition are commutative operations, meaning the order of the operands does not affect the outcome of the operation. So, when we write factorizations or partitions of numbers, we do not include variations that are the same set of integers. For example, $1+5$ and $5+1$ are the same partition.

We can visualize a partition using circles. To represent a partition of a number n, we arrange the n circles into rows. Each row represents a part in the partition. This is called a *Ferrers diagram*. For example here is a diagram of one partition of the number 11:



This represents the partition $5+3+2+1$ since the first row has 5 circles, the second has 3 circles, the third has 2 circles, and the fourth has 1 circle. We can easily find another partition of 11 by rotating this diagram by 90 degrees.



This diagram represents the partition $1+1+2+3+4$. These two partitions are said to be *conjugates* of one another.

We can ask many different questions about partitions. We could ask for all of the partitions of a number or some subset. For example, in our problem the partitions we considered were restricted to those that included 250. We could ask for partitions that did not include any duplicate parts or partitions that only included odd numbers. The possibilities are endless.