The CENTRE for EDUCATION in MATHEMATICS and COMPUTING
presents the

2013 Canadian Team Mathematics Contest (CTMC)

Individual Problems

1. A number is doubled and then 30 is added, giving a final result of 36. What was the original number?

2. In the showroom at the Centre for Exceptional Motor Cars, there are 19 cars. Each car has either 5 wheels or 6 wheels. If there are 100 wheels in total, how many 5-wheeled cars are there?

3. Find the area of the triangle formed by the lines $4x - 3y = 0, x + 2y = 11$ and the x-axis.

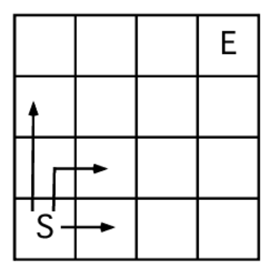
4. Two fair dice are tossed. One die has two sides painted green and four sides painted red. The second die has one side painted green and the other five sides painted purple. If the two dice are rolled, what is the probability that the two sides facing up are the same colour?

5. A sequence is created by beginning with $\longrightarrow$ and repeatedly turning the arrow 45° counter clockwise. The first terms of the sequence are
$\longrightarrow, \nearrow, \uparrow, \nwarrow, \longleftarrow, \swarrow, \ldots$. Determine the 2013th figure in the sequence.

6. Starting with the integer 7 and repeatedly doubling, we obtain
$14, 28, 56, 112, 224, 448, 896, 1792, \ldots$.
What is the largest integer less than 2013 that can be obtained by repeatedly doubling a positive integer less than 100?

7. Evaluate n given $\dfrac{1 + 2 + 3 + \ldots + (n-1) + n}{3n} = \dfrac{33}{2}$.

8. On the 4×4 grid of unit squares shown, a Canada goose walks from the square containing the letter S to the square containing the letter E using a sequence of moves. There are 3 possible moves that the Canada goose can make:



   A. It can go up 2 unit squares.
   B. It can go right 1 unit square.
   C. It can go up one unit square and then right 1 unit square.

   How many different paths from S to E make a sequence of these moves?

9. A cylindrical rain barrel has radius 50 cm and height 150 cm. A valve at the very bottom of the rain barrel leaks at a rate of 0.01 m^3/h. There is a hole **exactly** half way up the side of the barrel that leaks at a rate of 0.09 m^3/h. At the start of a storm, there is water at a depth of 20 cm in the rain barrel. If rainwater comes into the top of the rain barrel at a constant rate of 1 m^3/h, how long will it take before the rain barrel is completely full of water?
Give your answer in the form $\frac{m\pi}{n}$ where m and n are positive integers with no common divisor larger than 1.

10. Determine the smallest positive integer k for which there are positive integers a, b and c satisfying the three equations $k = 6a$, $k = 7b^2$ and $k = 8c^3$.