



# Canadian Mathematics Competition

An activity of the Centre for Education  
in Mathematics and Computing,  
University of Waterloo, Waterloo, Ontario

## Cayley Contest (Grade 10)

Wednesday, February 23, 2005

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**Time:** 60 minutes

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**Calculators are permitted.**

### Instructions

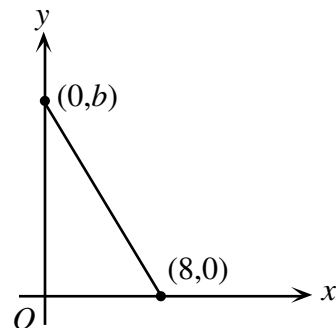
1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper left corner.
5. **Be certain that you code your name, age, sex, grade, and the Contest you are writing in the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.  
There is *no penalty* for an incorrect answer.  
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.

Scoring: There is *no penalty* for an incorrect answer.

Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

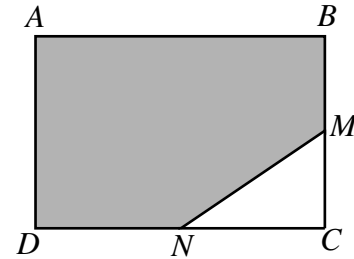
**Part A: Each correct answer is worth 5.**

- The expression  $a + 1 + a - 2 + a + 3 + a - 4$  is equal to  
(A)  $10a$       (B)  $0$       (C)  $4a - 2$       (D)  $4a + 2$       (E)  $-2a$
- The value of  $(\frac{4}{5})(\frac{5}{6})(\frac{6}{7})(\frac{7}{8})(\frac{8}{9})$  is  
(A)  $\frac{4}{9}$       (B)  $1$       (C)  $\frac{6}{7}$       (D)  $36$       (E)  $\frac{36}{25}$
- When 45 is divided by 7, the remainder is 3. What is the remainder when 70 is divided by 17?  
(A) 1      (B) 12      (C) 15      (D) 2      (E) 11
- If  $\frac{3}{x+10} = \frac{1}{2x}$ , then  $x$  equals  
(A)  $\frac{1}{2}$       (B) 10      (C)  $-4$       (D) 2      (E)  $-8$
- A teacher writes five different possible values for  $(5^2 - 4^2)^3$  on the board and asks her class to decide which is correct. The correct value is  
(A) 1      (B) 8      (C) 11 529      (D) 216      (E) 729
- Last week, a charity fundraiser had 8 volunteers who each worked 40 hours and who each raised \$18 per hour. This week, 12 volunteers, each working 32 hours, raised the same total amount of money. How much did each volunteer raise per hour this week?  
(A) \$9      (B) \$12      (C) \$15      (D) \$21      (E) \$24
- In the diagram, the line segment has slope  $-\frac{3}{2}$ . The value of  $b$  is  
(A) 10      (B) 12      (C) 6  
(D) 16      (E) 20



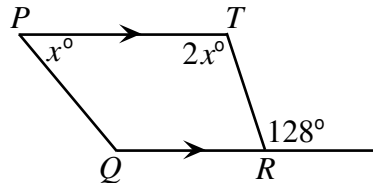
9.  $ABCD$  is a rectangle, with  $M$  the midpoint of  $BC$  and  $N$  the midpoint of  $CD$ . If  $CM = 4$  and  $NC = 5$ , what percent of the area of the rectangle is shaded?

(A) 70      (B) 78      (C) 80  
(D) 87.5      (E) 75



10. In the diagram,  $PT$  is parallel to  $QR$ . What is the size of  $\angle PQR$ ?

(A)  $116^\circ$       (B)  $168^\circ$       (C)  $138^\circ$   
(D)  $144^\circ$       (E)  $122^\circ$



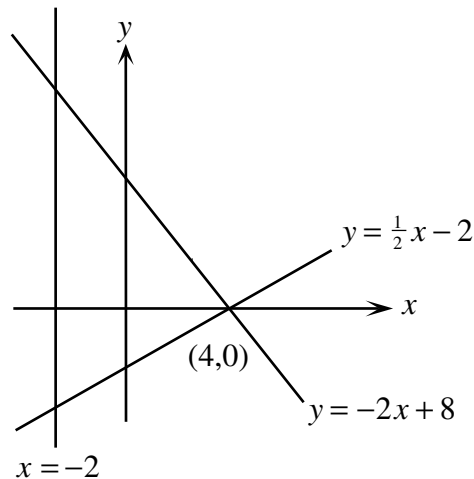
**Part B: Each correct answer is worth 6.**

11. During a football game, Matt kicked the ball three times. His longest kick was 43 metres and the three kicks averaged 37 metres. If the other two kicks were the same length, the distance, in metres, that each travelled was

(A) 31      (B) 37      (C) 35      (D) 34      (E) 36

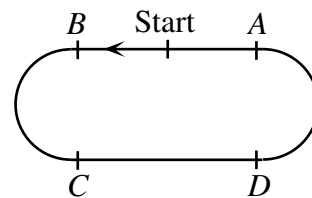
12. The lines  $y = -2x + 8$  and  $y = \frac{1}{2}x - 2$  meet at  $(4, 0)$ , as shown. The area of the triangle formed by these two lines and the line  $x = -2$  is

(A) 15      (B) 27      (C) 30  
(D) 36      (E) 45



13. A 400 m track is constructed so that the points  $A$ ,  $B$ ,  $C$ , and  $D$  divide the track into four segments of equal length. The Start is half-way between  $A$  and  $B$ . Andrew begins at the Start and walks at a steady rate of 1.4 m/s in a counter-clockwise direction. After exactly 30 minutes, to what point will Andrew be closest?

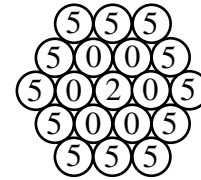
(A)  $A$       (B)  $B$       (C)  $C$   
(D)  $D$       (E) Start



14. If  $x$  is a *positive* integer less than 100, how many values of  $x$  make  $\sqrt{1 + 2 + 3 + 4 + x}$  an integer?

(A) 6      (B) 7      (C) 8      (D) 9      (E) 10

15. Starting with the 2 in the centre, the number 2005 can be formed by moving from circle to circle only if the two circles are touching. How many different paths can be followed to form 2005?



- (A) 36            (B) 24            (C) 12  
(D) 18            (E) 6

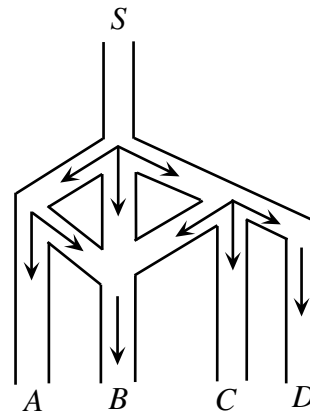
16. The *non-negative difference* between two numbers  $a$  and  $b$  is  $a - b$  or  $b - a$ , whichever is greater than or equal to 0. For example, the non-negative difference between 24 and 64 is 40. In the sequence 88, 24, 64, 40, 24,  $\dots$ , each number after the second is obtained by finding the non-negative difference between the previous 2 numbers. The sum of the first 100 numbers in this sequence is

- (A) 496            (B) 760            (C) 752            (D) 776            (E) 405

17.  $10^{100}$  is a googol.  $1000^{100}$  equals

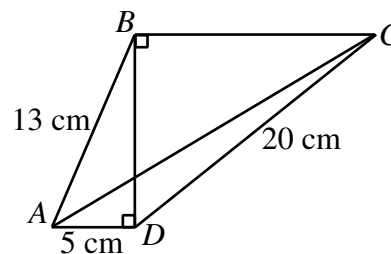
- (A) 100 googol                            (B) 3 googol                            (C) googol<sup>googol</sup>  
(D) googol<sup>2</sup>                                (E) googol<sup>3</sup>

18. Harry the Hamster is put in a maze, and he starts at point  $S$ . The paths are such that Harry can move forward only in the direction of the arrows. At any junction, he is equally likely to choose any of the forward paths. What is the probability that Harry ends up at  $B$ ?



- (A)  $\frac{2}{3}$             (B)  $\frac{13}{18}$             (C)  $\frac{11}{18}$   
(D)  $\frac{1}{3}$             (E)  $\frac{1}{4}$

19. In the diagram,  $AB = 13$  cm,  $DC = 20$  cm, and  $AD = 5$  cm. The length of  $AC$ , to the nearest tenth of a centimetre, is



- (A) 24.2            (B) 20.6            (C) 25.2  
(D) 23.4            (E) 24.9

20. There are 81 cars in the CMC parking lot, which are all Acuras, Beetles, or Camrys. There are half as many Acuras as Beetles. The number of Camrys is 80% of the number of Acuras and Beetles together. How many of the 81 cars are Beetles?

- (A) 36            (B) 30            (C) 45            (D) 51            (E) 66

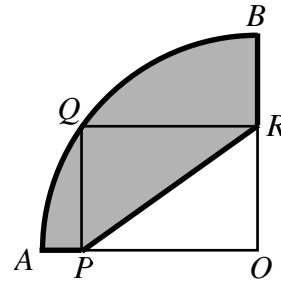
**Part C: Each correct answer is worth 8.**

21. In Yacleyland, the unit of money used is called the Yacley. There are only two denominations of paper money: the 17 Yacley bill and the 5 Yacley bill. How many different combinations of these bills total 453 Yacley?

(A) 3            (B) 4            (C) 5            (D) 6            (E) 7

22. In the diagram,  $AOB$  is a quarter circle of radius 10 and  $PQRO$  is a rectangle of perimeter 26. The perimeter of the shaded region is

(A)  $7 + 5\pi$     (B)  $13 + 5\pi$     (C)  $17 + 5\pi$   
 (D)  $7 + 25\pi$     (E)  $17 + 25\pi$

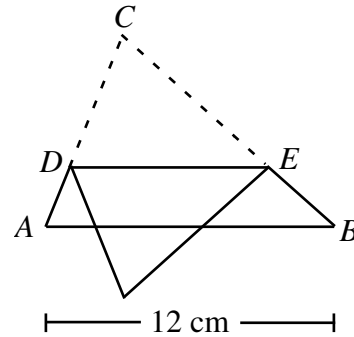


23. At 12:00 noon, Anna and Bill left home and walked in the same direction. Anna walked at 4 km/h and Bill walked at 3 km/h. At 12:15 their dog Dexter, running at 6 km/h, left home to run after them. The dog ran until it caught up to Anna, then it ran back to Bill. (In his excitement, Dexter lost no time in turning around once he reached Anna.) At what time did Bill meet Dexter on Dexter's way back?

(A) 1:00 p.m.    (B) 1:15 p.m.    (C) 12:45 p.m.    (D) 1:05 p.m.    (E) 12:50 p.m.

24. The base of a triangular piece of paper  $ABC$  is 12 cm long. The paper is folded down over the base, with the crease  $DE$  parallel to the base of the paper. The area of the triangle that projects below the base is 16% that of the area of the triangle  $ABC$ . The length of  $DE$ , in cm, is

(A) 9.6            (B) 8.4            (C) 7.2  
 (D) 4.8            (E) 6.96



25. The positive integers  $a, b$  and  $c$  satisfy  $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$ .

The sum of all possible values of  $a \leq 100$  is

(A) 315            (B) 615            (C) 680            (D) 555            (E) 620



## Canadian Mathematics Competition



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