Canadian Mathematics Competition

An activity of The Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

2004 Solutions

Gauss Contest

(Grades 7 and 8)

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Part A

1. Simplifying,

$$\frac{10+20+30+40}{10}=\frac{100}{10}=10$$

Answer: (C)

2. Using a common denominator,

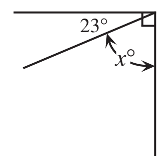
$\frac{1}{2}-\frac{1}{8}=\frac{4}{8}-\frac{1}{8}=\frac{3}{8}$

Answer: (A)

3. Seven thousand twenty-two is $7000+22=7022$.

Answer: (D)

4. From the diagram, $23^\circ+x^\circ=90^\circ$ so $x^\circ=67^\circ$ or $x=67$.



Answer: (C)

5. Since Sally was 7 years old five years ago, then she is 12 years old today. Thus, in two more years, she will be 14.

Answer: (B)

6. Since Stuart earns 5 reward points for every \$25 he spends, then when he spends \$200, he earns $\frac{200}{25}\times 5=8\times 5=40$ points.

Answer: (C)

7. Using a calculator, $\frac{8}{9}=0.888...$, $\frac{7}{8}=0.875$, $\frac{66}{77}=\frac{6}{7}=0.857...$, $\frac{55}{66}=\frac{5}{6}=0.833...$, $\frac{4}{5}=0.8$, so $\frac{8}{9}=0.888...$ is the largest.

Answer: (A)

8. There are 6 balls in the box. 5 of the balls in the box are not grey. Therefore, the probability of selecting a ball that is not grey is $\frac{5}{6}$.

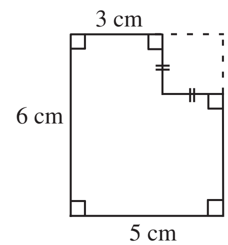
Answer: (E)

9. The sum of the numbers in the second column is $19+15+11=45$, so the sum of the numbers in any row column or diagonal is 45. The sum of the two numbers already in the first row is 33, so the third number in the first row (in the upper right corner) must be 12. Finally, the diagonal from bottom left to top right has x, 15 and 12, so $x+15+12=45$ or $x=18$.

Answer: (E)

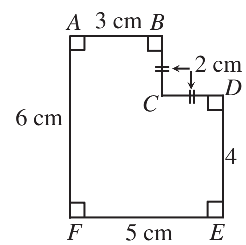
10. *Solution 1*

We notice that if we complete the given figure to form a rectangle, then the perimeter of this rectangle and the original figure are identical. Therefore, the perimeter is $2 \times 5 + 2 \times 6 = 22$ cm.



Solution 2

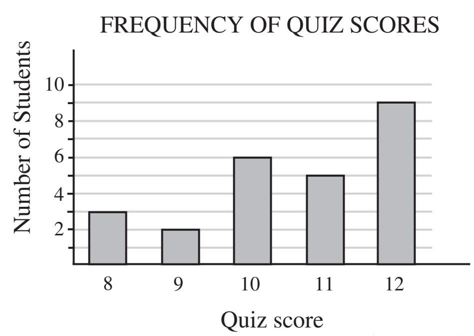
Since the width of the figure is 5 cm, then
$AB + CD = 5$ cm, so $CD = BC = 2$ cm.
Since the height of the figure is 6 cm, then
$BC + DE = 6$ cm, so $DE = 4$ cm.
Therefore, the perimeter is $3 + 2 + 2 + 4 + 5 + 6 = 22$ cm.



Answer: (E)

Part B

11. When we list the quiz scores in ascending order, including repetition, we get 8, 8, 8, 9, 9, 10, 10, 10, 10, 10, 10, 11, 11, 11, 11, 11, 12, 12, 12, 12, 12, 12, 12, 12, 12.
Since there are 25 scores, the middle score is the 13th along, so the median is 11.



Answer: (D)

12. In travelling between the two lakes, the total change in elevation is $174.28 - 75.00 = 99.28$ m.
Since this change occurs over 8 hours, the average change in elevation per hour is
$\frac{99.28 \text{ m}}{8 \text{ h}} = 12.41 \text{ m / h}$.

Answer: (A)

13. We make a chart of the pairs of positive integers which sum to 11 and their corresponding products:

First integer	Second integer	Product
1	10	10
2	9	18
3	8	24
4	7	28
5	6	30

so the greatest possible product is 30.

Answer: (E)

14. Evaluating the exponents, $3^2 = 9$ and $3^3 = 27$, so the even whole numbers between the two given numbers are the even whole numbers from 10 to 26, inclusive. These are 10, 12, 14, 16, 18, 20, 22, 24, and 26, so there are 9 of them.

Answer: (A)

15. If $P = 1000$ and $Q = 0.01$, then

$$P + Q = 1000 + 0.01 = 1000.01$$
$$P \times Q = 1000 \times 0.01 = 10$$
$$\frac{P}{Q} = \frac{1000}{0.01} = 100000$$
$$\frac{Q}{P} = \frac{0.01}{1000} = 0.00001$$
$$P - Q = 1000 - 0.01 = 999.99$$

so the largest is $\frac{P}{Q}$.

Answer: (C)

16. The volume of the box of $40 \times 60 \times 80 = 192000\ \text{cm}^3$. The volume of each of the blocks is $20 \times 30 \times 40 = 24000\ \text{cm}^3$. Therefore, the maximum number of blocks that can fit inside the box is $\frac{192000\ \text{cm}^3}{24000\ \text{cm}^3} = 8$. 8 blocks can indeed be fit inside this box. Can you see how?

Answer: (D)

17 In the recipe, the ratio of volume of flour to volume of shortening is 5 : 1. Since she uses $\frac{2}{3}$ cup of shortening, then to keep the same ratio as called for in the recipe, she must use $5 \times \frac{2}{3} = \frac{10}{3} = 3\frac{1}{3}$ cups of flour.

Answer: (B)

18. The rectangular prism in the diagram is made up of 12 cubes. We are able to see 10 of these 12 cubes. One of the two missing cubes is white and the other is black. Since the four blocks of each colour are attached together to form a piece, then the middle block in the back row in the bottom layer must be white, so the missing black block is the leftmost block of the back row in the bottom layer. Thus, the leftmost block in the back row in the top layer is attached to all three of the other black blocks, so the shape of the black piece is (A). (This is the only one of the 5 possibilities where one block is attached to three other blocks.)

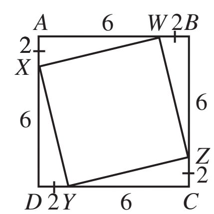
Answer: (A)

19. Since the number is divisible by $8 = 2^3$, by $12 = 2^2 \times 3$, and by $18 = 2 \times 3^2$, then the number must have at least three factors of 2 and two factors of 3, so the number must be divisible by $2^3 \times 3^2 = 72$. Since the number is a two-digit number which is divisible by 72, it must be 72 (it cannot have more than two digits), so it is between 60 and 79.

Answer: (D)

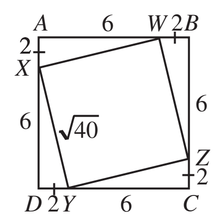
20. *Solution 1*

Since the area of square *ABCD* is 64, then the side length of square *ABCD* is 8. Since $AX = BW = CZ = DY = 2$, then $AW = BZ = CY = DX = 6$. Thus, each of triangles *XAW*, *WBZ*, *ZCY* and *YDX* is right-angled with one leg of length 2 and the other of length 6. Therefore, each of these four triangles has area $\frac{1}{2}(2)(6) = 6$. Therefore, the area of square *WXYZ* is equal to the area of square *ABCD* minus the sum of the areas of the four triangles, or $64 - 4(6) = 40$.



Solution 2

Since the area of square *ABCD* is 64, then the side length of square *ABCD* is 8. Since $AX = BW = CZ = DY = 2$, then $AW = BZ = CY = DX = 6$.
By the Pythagorean Theorem,
$XW = WZ = ZY = YX = \sqrt{2^2 + 6^2} = \sqrt{4 + 36} = \sqrt{40}$.
Therefore, the area of square *WXYZ* is $\left(\sqrt{40}\right)^2 = 40$.



Answer: (D)

Part C

21. In the diagram, we will refer to the horizontal dimension as the width of the room and the vertical dimension as the length of the room. Since the living room is square and has an area of 16 m^2, then it has a length of 4 m and a width of 4 m. Since the laundry room is square and has an area of 4 m^2, then it has a length of 2 m and a width of 2 m. Since the dining room has a length of 4 m (the same as the length of the living room) and an area of 24 m^2, then it has a width of 6 m. Thus, the entire ground floor has a width of 10 m, and so the kitchen has a width of 8 m (since the width of the laundry room is 2 m) and a length of 2 m(since the length of the laundry room is 2 m), and so the kitchen has an area of 16 m^2.

Answer: (B)

22. Let the volume of a large glass be *L* and of a small glass be *S*. Since the jug can exactly fill either 9 small glasses and 4 large glasses, or 6 small glasses and 6 large glasses, then $9S + 4L = 6S + 6L$ or $3S = 2L$. In other words, the volume of 3 small glasses equals the volume of 2 large glasses. (We can also see this without using algebra – if we compare the two cases, we can see that if we remove 3 small glasses then we increase the volume by 2 large glasses.) Therefore, the volume of 9 small glasses equals the volume of 6 large glasses. Thus, the volume of 9 small glasses and 4 large glasses equals the volume of 6 large glasses and 4 large glasses, or 10 large glasses in total, and so the jug can fill 10 large glasses in total.

Answer: (C)

23. In her 40 minutes (or $\frac{2}{3}$ of an hour) on city roads driving at an average speed of 45 km/h, Sharon drives $\left(\frac{2}{3}\text{ h}\right)\times(45\text{ km / h}) = 30\text{ km}$. So the distance that she drives on the highway must be $59\text{ km} - 30\text{ km} = 29\text{ km}$. Since she drives this distance in 20 minutes (or $\frac{1}{3}$ of an hour), then her average speed on the highway is $\dfrac{29\text{ km}}{\frac{1}{3}\text{ h}} = (29\times 3)\text{ km / h} = 87\text{ km / h}$.

Answer: (C)

24. We consider each possible number of silver medals starting with 8.
Could she have won 8 silver medals? This would account for 24 points in 8 events, but since she won 27 points in 8 events, this is not possible.
Could she have won 7 silver medals? This would account for 21 points in 7 events, and so in the remaining 1 event, she would have won 6 points, which is impossible, since she could not score more than 5 points (a gold medal) on this event.
Could she have won 6 silver medals? This would account for 18 points in 6 events, and so in the remaining 2 events, she would have won 9 points, which is impossible, since we cannot combine either two 5s, two 1s or a 1 and a 5 to get 9.
Could she have won 5 silver medals? This would account for 15 points in 5 events, and so in the remaining 3 events, she would have won 12 points, which is impossible. (Try combining up to three 5s and 1s to get 12. We need at least two 5s and two 1s to make 12.)
Could she have won 4 silver medals? This would account for 12 points in 4 events, and so in the remaining 4 events, she would have won 15 points. This is possible – she could win gold on 3 of the 4 remaining events (for 15 points in total) and no medal on the last event (there are 6 competitors and only 3 medals for each event, so there are competitors who do not win medals).
Thus, the maximum number of silver medals she could have won is 4.

Answer: (D)

Answer: (D)

25. *Solution 1*
Start with a grid with two columns and ten rows. There are 10 ways to place the domino horizontally (one in each row) and 18 ways to place the column vertically (nine in each column), so 28 ways overall.
How many more positions are added when a new column is added? When a new column is added, there are 9 new vertical positions (since the column has ten squares) and 10 new horizontal positions (one per row overlapping the new column and the previous column). So there are 19 new positions added.
How many times do we have to add 19 to 28 to get to 2004? In other words, how many times does 19 divide into $2004 - 28 = 1976$? Well, $1976 \div 19 = 104$, so we have to add 104 new columns to the original 2 columns, for 106 columns in total.

Solution 2
Let the number of columns be n.
In each column, there are 9 positions for the domino (overlapping squares 1 and 2, 2 and 3, 3 and 4, and so on, down to 9 and 10).
In each row, there are $n-1$ positions for the domino (overlapping squares 1 and 2, 2 and 3, 3 and 4, and so on, along to $n-1$ and n).
Therefore, the total number of positions for the domino equals the number of rows times the number of positions per row plus the number of columns times the number of positions per column, or $n(9)+10(n-1)=19n-10$. We want this to equal 2004, so $19n-10=2004$ or $19n=2014$ or $n=106$. Thus, there are 106 columns.

Answer: (B)