Part A

1. The value of $\frac{1}{2}+\frac{1}{4}$ is

   (**A**) 1 (**B**) $\frac{1}{8}$ (**C**) $\frac{1}{6}$ (**D**) $\frac{2}{6}$ (**E**) $\frac{3}{4}$

   Solution
   Using a common denominator,
   $\frac{1}{2}+\frac{1}{4}=\frac{2}{4}+\frac{1}{4}=\frac{3}{4}$. Answer: (E)

2. The expression $6 ¥ 1000 + 5 ¥ 100 + 6 ¥ 1$ is equivalent to
   (**A**) 656 (**B**) 6506 (**C**) 6056 (**D**) 60 506 (**E**) 6560

   Solution
   Expanding,
   $6 ¥ 1000 + 5 ¥ 100 + 6 ¥ 1 = 6000 + 500 + 6 = 6506$. Answer: (B)

3. The value of $3^2-(4 ¥ 2)$ is
   (**A**) 4 (**B**) 17 (**C**) 1 (**D**) –2 (**E**) 0

   Solution
   By order of operations,
   $3^2-(4 ¥ 2)=9-(4 ¥ 2)=9-8=1$. Answer: (C)

4. An integer is divided by 7 and the remainder is 4. An example of such an integer is
   (**A**) 14 (**B**) 15 (**C**) 16 (**D**) 17 (**E**) 18

   Solution
   Since 14 is a multiple of 7, then 18 (which is 4 more than 14) gives a remainder of 4 when divided by 7. Answer: (E)

5. Which of the following expressions is equal to an odd integer?

   (**A**) $3(5)+1$ (**B**) $2(3+5)$ (**C**) $3(3+5)$ (**D**) $3+5+1$ (**E**) $\frac{3+5}{2}$

   Solution
   Evaluating the choices,

   (**A**) $3(5)+1=16$ (**B**) $2(3+5)=16$ (**C**) $3(3+5)=24$ (**D**) $3+5+1=9$ (**E**) $\frac{3+5}{2}=4$

   Choice **(D)** gives the only odd integer. Answer: (D)

6. Qaddama is 6 years older than Jack. Jack is 3 years younger than Doug. If Qaddama is 19 years old, how old is Doug?
(**A**) 17 (**B**) 16 (**C**) 10 (**D**) 18 (**E**) 15

Solution
If Qaddama is 6 years older than Jack and she is 19 years old, then Jack is 13 years old. If Jack is 3 years younger than Doug, then Doug must be 16 years of age. ANSWER: (B)

7. The volume of a rectangular box is 144 cm^3. If its length is 12 cm and its width is 6 cm, what is its height?
(**A**) 126 cm (**B**) 72 cm (**C**) 4 cm (**D**) 8 cm (**E**) 2 cm

Solution
We know that $\text{Volume} = \text{Length} \times \text{Width} \times \text{Height}$, the volume is 144 cm^3, and $\text{Length} \times \text{Width} = 72\text{ cm}^2$. Thus, $144\text{ cm}^3 = 72\text{ cm}^2 \times \text{Height}$, or $\text{Height} = 2\text{ cm}$.
ANSWER: (E)

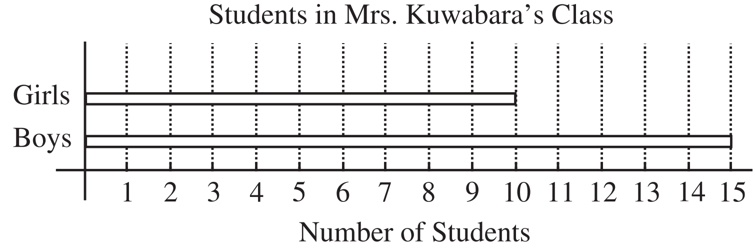
8. In a jar, the ratio of the number of oatmeal cookies to the number of chocolate chip cookies is 5:2. If there are 20 oatmeal cookies, the number of chocolate chip cookies in the jar is
(**A**) 28 (**B**) 50 (**C**) 8 (**D**) 12 (**E**) 18

Solution
The ratio 5:2 indicates that there are 5 oatmeal cookies for every 2 chocolate chip cookies. Since there are 20 oatmeal cookies, there are four groups of 5 oatmeal cookies. Thus there are $4 \times 2 = 8$ chocolate chip cookies.

Algebraically, we could let x represent the number of chocolate chip cookies. Then $5:2 = 20:x$, or $\frac{5}{2} = \frac{20}{x}$. If we want to write $\frac{5}{2}$ as a fraction with a numerator of 20, we multiply both the numerator and denominator by 4, ie. $\frac{5}{2} = \frac{5 \times 4}{2 \times 4} = \frac{20}{8}$. Therefore, $x = 8$. ANSWER: (C)

9. The bar graph shows the numbers of boys and girls in Mrs. Kuwabara's class. The percentage of students in the class who are girls is
(**A**) 40% (**B**) 15% (**C**) 25% (**D**) 10% (**E**) 60%

Students in Mrs. Kuwabara's Class



Solution

From the graph, there are 10 girls and 15 boys in the class. Then, there are 25 students in total in the class, so the percentage of girls is $\frac{10}{25}$ ¥ $100\% = 40\%$. ANSWER: (A)

10. Which of the following statements is **not** true?
(**A**) A quadrilateral has four sides.
(**B**) The sum of the angles in a triangle is 180∂.
(**C**) A rectangle has four 90∂ angles.
(**D**) A triangle can have two 90∂ angles.
(**E**) A rectangle is a quadrilateral.

Solution

A quadrilateral has four sides, by definition.
The sum of the angles in a triangle is 180°.
A rectangle has four 90° angles, by definition.
A rectangle is a quadrilateral, since it has four sides.
However, a triangle cannot have two 90°, since its three angles add to 180°, and its third angle cannot be 0°. ANSWER: (D)

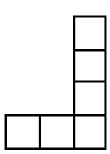
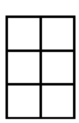
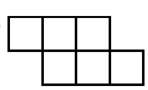
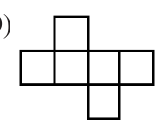
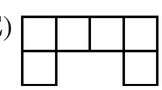
Part B

11. A palindrome is a positive integer whose digits are the same when read forwards or backwards. 2002 is a palindrome. What is the smallest number which can be added to 2002 to produce a larger palindrome?
(**A**) 11 (**B**) 110 (**C**) 108 (**D**) 18 (**E**) 1001

Solution

The best way to analyze this problem is by asking the question, "What is the next palindrome bigger than 2002?" Since the required palindrome should be of the form $2aa2$, where the middle two digits (both a) do not equal 0, it must be the number 2112. Thus, the number that must be added to 2002 is $2112 - 2002 = 110$. ANSWER: (B)

12. Which of the following can be folded along the lines to form a cube?

(**A**)  (**B**)  (**C**)  (**D**)  (**E**) 

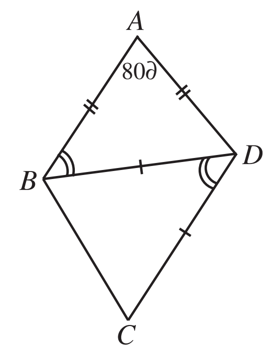
Solution

Only choice (**D**) can be folded to form a cube. (Try constructing these nets to check this answer.)
ANSWER: (D)

13. If $a+b=12$, $b+c=16$, and $c=7$, what is the value of a?
(**A**) 1 (**B**) 5 (**C**) 9 (**D**) 7 (**E**) 3

Solution
Since $c=7$ and $b+c=16$, then $b+7=16$, or $b=9$.
Since $b=9$ and $a+b=12$, then $a+9=12$, or $a=3$. ANSWER: (E)

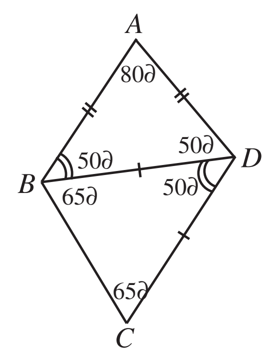
14. In the diagram, $\angle ABD = \angle BDC$ and $\angle DAB = 80^\circ$. Also, $AB = AD$ and $DB = DC$. The measure of $\angle BCD$ is
(**A**) 65° (**B**) 50° (**C**) 80°
(**D**) 60° (**E**) 70°



Solution
Since $\triangle ABD$ is isosceles, then $\angle ABD = \angle ADB$.
Therefore,

$$80^\circ + \angle ABD + \angle ADB = 180^\circ$$
$$2 \times \angle ABD = 100^\circ \quad (\text{since } \angle ABD = \angle ADB)$$
$$\angle ABD = 50^\circ$$

Thus, $\angle BDC = 50^\circ$ as well, since $\angle ABD = \angle BDC$. Since $\triangle BDC$ is also isosceles, then if we repeat a similar calculation to above, we obtain that $\angle BCD = 65^\circ$.



ANSWER: (A)

15. A perfect number is an integer that is equal to the sum of all of its positive divisors, except itself. For example, 28 is a perfect number because $28 = 1+2+4+7+14$. Which of the following is a perfect number?
(**A**) 10 (**B**) 13 (**C**) 6 (**D**) 8 (**E**) 9

Solution
We must check each of the answers:

	Number	Positive divisors	Sum of all positive divisors except itself
(A)	10	1, 2, 5, 10	$1+2+5=8$
(B)	13	1, 13	1
(C)	6	1, 2, 3, 6	$1+2+3=6$
(D)	8	1, 2, 4, 8	$1+2+4=7$
(E)	9	1, 3, 9	$1+3=4$

The only number from this set that is a perfect number is 6. (Note that the next two perfect number bigger than 28 are 496 and 8128.) Answer: (C)

16. Three pennies are flipped. What is the probability that they all land with heads up?

(**A**) $\frac{1}{8}$ (**B**) $\frac{1}{6}$ (**C**) $\frac{1}{4}$ (**D**) $\frac{1}{3}$ (**E**) $\frac{1}{2}$

Solution

If we toss one penny, the probability that it lands with heads up is $\frac{1}{2}$.

Since we want three heads up, we must multiply these probabilities together. That is, the probability is $\frac{1}{2}$ ¥ $\frac{1}{2}$ ¥ $\frac{1}{2} = \frac{1}{8}$.

Alternatively, we could list all of the possibilities for the 3 pennies, using H to represent heads and T to represent tails:

HHH	THH
HHT	THT
HTH	TTH
HTT	TTT

This means that there are 8 equally likely possibilities, one of which is the desired possibility.

Therefore, the probability of three heads coming up is $\frac{1}{8}$. ANSWER: (A)

17. If P is a negative integer, which of the following is always positive?

(**A**) P^2 (**B**) $\frac{1}{P}$ (**C**) $2P$ (**D**) $P-1$ (**E**) P^3

Solution

If we try $P = -1$,

(**A**) $P^2 = 1$ (**B**) $\frac{1}{P} = -1$ (**C**) $2P = -2$ (**D**) $P - 1 = -2$ (**E**) $P^3 = -1$

and so the only possibility for the correct answer is (**A**). (In fact, P^2 is always greater than or equal to 0, regardless of the choice for P.) ANSWER: (A)

18. When expanded, the number of zeros in 1000^{10} is

(**A**) 13 (**B**) 30 (**C**) 4 (**D**) 10 (**E**) 1000

Solution

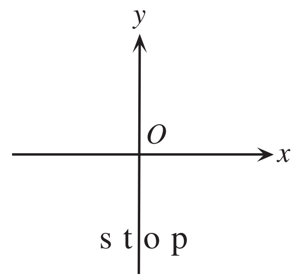
Using exponent laws,

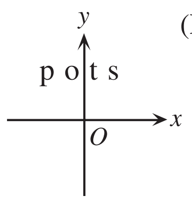
$$1000^{10} = \left(10^3\right)^{10} = 10^{3¥10} = 10^{30}$$

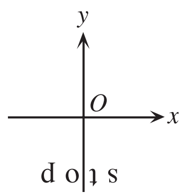
so if we were to write the number out in full, there should be 30 zeros.

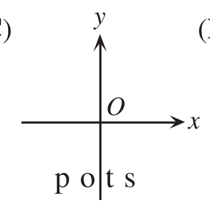
ANSWER: (B)

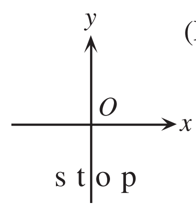
19. The word "stop" starts in the position shown in the diagram to the right. It is then rotated 180∂ clockwise about the origin, O, and this result is then reflected in the x-axis. Which of the following represents the final image?

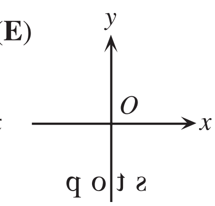


(**A**)


(**B**)


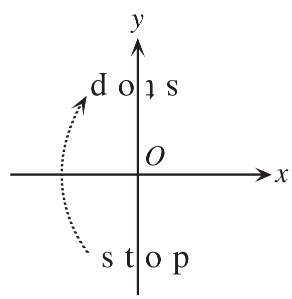
(**C**)


(**D**)


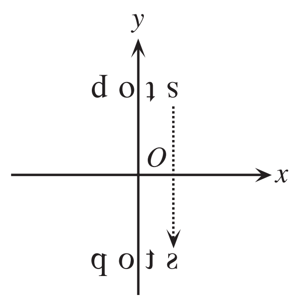
(**E**)


Solution

If we start by rotating by 180° and then reflecting that image, we would get the following:



Rotation of 180°



Reflection in x-axis

Answer: (E)

20. The units digit (that is, the last digit) of 7^{62} is

(**A**) 7 (**B**) 1 (**C**) 3 (**D**) 9 (**E**) 5

Solution

If we write out the first few powers of 7,

$$7^1 = 7,\ 7^2 = 49,\ 7^3 = 343,\ 7^4 = 2401,\ 7^5 = 16\,807, \ldots$$

we can see that the units digit follows the pattern 7, 9, 3, 1, 7, 9, 3, 1, 7, … (That is to say, the units digit of a product depends only on the units digits of the numbers being multiplied together. This tells us that we only need to look at the units digit of the previous power to determine the units digit of a given power.)

So the pattern 7, 9, 3, 1, repeats in blocks of four. Since 60 is a multiple of 4, this means that 7^{60} has a units digit of 1, and so 7^{62} has a units digit of 9. Answer: (D)

Part C

21. A rectangle has sides of integer length (when measured in cm) and an area of 36 cm^2. What is the maximum possible perimeter of the rectangle?
(**A**) 72 cm (**B**) 80 cm (**C**) 26 cm (**D**) 74 cm (**E**) 48 cm

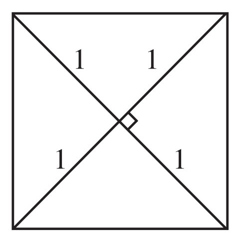
Solution
Since the area is 36 cm^2 and the sides have integer length, then we make a table of the possibilities:

Side lengths	Perimeter
1, 36	$2(1+36)=74$
2, 18	$2(2+18)=40$
3, 12	$2(3+12)=30$
4, 9	$2(4+9)=26$
6, 6	$2(6+6)=24$

So the maximum possible perimeter is 74 cm. ANSWER: (D)

22. If each diagonal of a square has length 2, then the area of the square is
(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 5

Solution
We draw the square and its two diagonals.
The diagonals of a square cut each other into two equal parts, and intersect at right angles. So we can decompose the square into 4 identical triangles with base 1 and height 1. So the area of the square is $4\left[\frac{1}{2}(1)(1)\right]=4\left[\frac{1}{2}\right]=2$.



ANSWER: (B)

23. A map is drawn to a scale of 1:10 000. On the map, the Gauss Forest occupies a rectangular region measuring 10 cm by 100 cm. What is the actual area of the Gauss Forest, in km^2?
(**A**) 100 (**B**) 1 000 000 (**C**) 1000 (**D**) 1 (**E**) 10

Solution
The actual lengths of the sides of the Gauss Forest are 10 000 times the lengths of the sides on the map. So the one side has length

$$10\ 000 \times 10\text{ cm} = 100\ 000\text{ cm} = 1000\text{ m} = 1\text{ km}$$

and the other side has length

$$10\ 000 \times 100\text{ cm} = 1\ 000\ 000\text{ cm} = 10\ 000\text{ m} = 10\text{ km}.$$

The actual area of the Gauss Forest is therefore $1\text{ km} \times 10\text{ km} = 10\text{ km}^2$.

ANSWER: (E)

24. Veronica has 6 marks on her report card.
The mean of the 6 marks is 74.
The mode of the 6 marks is 76.
The median of the 6 marks is 76.
The lowest mark is 50.
The highest mark is 94.
Only one mark appears twice and no mark appears more than twice.
Assuming all of her marks are integers, the number of possibilities for her second lowest mark is
(**A**) 17 (**B**) 16 (**C**) 25 (**D**) 18 (**E**) 24

Solution

Since the mode of Veronica's 6 marks is 76, and only one mark appears more than once (and no marks appear more than twice), then two of the marks must be 76. This tells us that four of her marks were 50, 76, 76, 94.

Since the median of her marks is 76 and she has six marks in total (that is, an *even* number of marks), then the two marks of 76 must be 3rd and 4th when the marks are arranged in increasing order.

Let the second lowest mark be M, and the second highest be N. So the second lowest mark M is between (but not equal to) 50 and 76, and the second highest mark N is between (but not equal to) 76 and 94.

We still need to use the fact that the mean of Veronica's marks is 74, so

$$\begin{aligned}\frac{50 + M + 76 + 76 + N + 94}{6} &= 74 \\ M + N + 296 &= 444 \\ M + N &= 148 \\ M &= 148 - N \quad (*)\end{aligned}$$

We know already that M is one of 51 through 75, but the possibilities for N and the equation (*) restrict these possibilities further.

Since N can be any of 77 through 93, there are exactly 17 possibilities for N. The largest value of M corresponds to $N = 77$ (ie. $M = 71$) and the smallest value for M is when $N = 93$ (ie. $M = 55$). Thus the possibilities for M are 55 through 71, ie. there are 17 possibilities in total for M, the second smallest mark.

ANSWER: (A)

25. Emily has created a jumping game using a straight row of floor tiles that she has numbered 1, 2, 3, 4, … . Starting on tile 2, she jumps along the row, landing on every second tile, and stops on the second last tile in the row. Starting from this tile, she turns and jumps back toward the start, this time landing on every third tile. She stops on tile 1. Finally, she turns again and jumps along the row, landing on every fifth tile. This time, she again stops on the second last tile. The number of tiles in the row could be
(**A**) 39 (**B**) 40 (**C**) 47 (**D**) 49 (**E**) 53

Solution

Since Emily first starts on tile 2 and jumps on every second tile, then she lands only on even numbered tiles. Since she stops on the second last tile, the total number of tiles is odd.

Next, Emily jumps back along the row by 3's and ends on tile 1. So every tile that she lands on this time has a number which is 1 more than a multiple of 3 (eg. 1, 4, 7, etc.) So the second last tile has a number that is 1 more than a multiple of 3. This tells us that the overall number of tiles in the row is 2 more than a multiple of 3.

These two conditions tell us that the total number of tiles cannot be 39, 40 or 49.

Lastly, Emily jumps by 5's along the row starting at 1. This says each tile that she lands on has a number that is 1 more than a multiple of 5. By the same reasoning as above, the total number of tiles in the row is 2 more than a multiple of 5.

Of the two remaining possibilities (47 and 53), the only one that satisfies this last condition is 47, and so 47 satisfies all 3 of the required conditions.

(Work back through Emily's steps using the fact that she starts with 47 tiles to check that this does work.) ANSWER: (C)

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