Canadian Mathematics Competition

An activity of The Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

2002 Solutions

Gauss Contest

(Grades 7 and 8)

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Part A

1. When the numbers 8, 3, 5, 0, 1 are arranged from smallest to largest, the middle number is
(**A**) 5 (**B**) 8 (**C**) 3 (**D**) 0 (**E**) 1

Solution
If we rearrange the given numbers from smallest to largest, we would have 0, 1, 3, 5, 8.
The middle number is 3. ANSWER: (C)

2. The value of $0.9 + 0.99$ is
(**A**) 0.999 (**B**) 1.89 (**C**) 1.08 (**D**) 1.98 (**E**) 0.89

Solution
Adding,

$$\begin{array}{r} 0.9 \\ +\ 0.99 \\ \hline 1.89 \end{array}$$

ANSWER: (B)

3. $\dfrac{2+1}{7+6}$ equals
(**A**) $\frac{3}{13}$ (**B**) $\frac{21}{76}$ (**C**) $\frac{1}{21}$ (**D**) $\frac{2}{13}$ (**E**) $\frac{1}{14}$

Solution
Evaluating,
$\dfrac{2+1}{7+6} = \dfrac{3}{13}$. ANSWER: (A)

4. 20% of 20 is equal to
(**A**) 400 (**B**) 100 (**C**) 5 (**D**) 2 (**E**) 4

Solution
20% of 20 equals 0.2 ¥ 20 = 4. Alternatively, 20% of 20 is $\frac{1}{5}$ of 20, or 4.
ANSWER: (E)

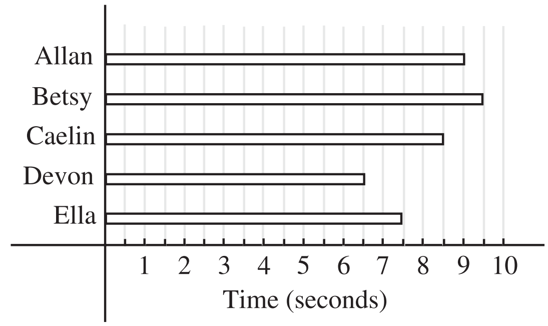
5. Tyesha earns \$5 per hour babysitting, and babysits for 7 hours in a particular week. If she starts the week with \$20 in her bank account, deposits all she earns into her account, and does not withdraw any money, the amount she has in her account at the end of the week is
(**A**) \$35 (**B**) \$20 (**C**) \$45 (**D**) \$55 (**E**) \$65

Solution

If Tyesha earns \$5 per hour and works for 7 hours, then she earns 7 ¥ \$5 = \$35 in total. If she started with \$20 in her bank account and adds the \$35, she will have \$20 + \$35 = \$55 in her account.
ANSWER: (D)

6. Five rats competed in a 25 metre race. The graph shows the time that each rat took to complete the race. Which rat won the race?

(**A**) Allan (**B**) Betsy (**C**) Caelin
(**D**) Devon (**E**) Ella



Solution

Since each of the rats completed the race, then the rat taking the least amount of time won the race. Since Devon took the least amount of time, she was the winner.
ANSWER: (D)

7. The mean (average) of the numbers 12, 14, 16, and 18, is

(**A**) 30 (**B**) 60 (**C**) 17 (**D**) 13 (**E**) 15

Solution

The mean of the given numbers is

$$\frac{12+14+16+18}{4} = \frac{60}{4} = 15.$$

ANSWER: (E)

8. If $P = 1$ and $Q = 2$, which of the following expressions is **not** equal to an integer?

(**A**) $P + Q$ (**B**) P ¥ Q (**C**) $\frac{P}{Q}$ (**D**) $\frac{Q}{P}$ (**E**) P^Q

Solution

Evaluating the choices,

(A) $P + Q = 3$ **(B)** P ¥ $Q = 2$ **(C)** $\frac{P}{Q} = \frac{1}{2}$ **(D)** $\frac{Q}{P} = \frac{2}{1} = 2$ **(E)** $P^Q = 1^2 = 1$

ANSWER: (C)

9. Four friends equally shared $\frac{3}{4}$ of a pizza, which was left over after a party. What fraction of a whole pizza did each friend get?

(**A**) $\frac{3}{8}$ (**B**) $\frac{3}{16}$ (**C**) $\frac{1}{12}$ (**D**) $\frac{1}{16}$ (**E**) $\frac{1}{8}$

Solution

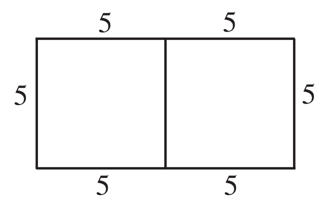
If $\frac{3}{4}$ of a pizza was shared by 4 friends, they would each receive $\frac{1}{4}$ of $\frac{3}{4}$, or $\frac{1}{4}¥\frac{3}{4}=\frac{3}{16}$ of the pizza.

ANSWER: (B)

10. Two squares, each with an area of 25 cm^2, are placed side by side to form a rectangle. What is the perimeter of this rectangle?
(**A**) 30 cm (**B**) 25 cm (**C**) 50 cm (**D**) 20 cm (**E**) 15 cm

Solution

If the two squares are placed side by side, the rectangle shown would be formed.
The perimeter of this newly formed rectangle is 30 cm.



ANSWER: (A)

Part B

11. After running 25% of a race, Giselle had run 50 metres. How long was the race, in metres?
(**A**) 100 (**B**) 1250 (**C**) 200 (**D**) 12.5 (**E**) 400

Solution

If 25% of a race is 50 metres, then 100% of the race is $\frac{100}{25}¥50=200$ metres.

ANSWER: (C)

12. Qaddama is 6 years older than Jack. Jack is 3 years younger than Doug. If Qaddama is 19 years old, how old is Doug?
(**A**) 17 (**B**) 16 (**C**) 10 (**D**) 18 (**E**) 15

Solution

If Qaddama is 6 years older than Jack and she is 19 years old, then Jack is 13 years old. If Jack is 3 years younger than Doug, then Doug must be 16 years of age. ANSWER: (B)

13. A palindrome is a positive integer whose digits are the same when read forwards or backwards. For example, 2002 is a palindrome. What is the smallest number which can be added to 2002 to produce a larger palindrome?
(**A**) 11 (**B**) 110 (**C**) 108 (**D**) 18 (**E**) 1001

Solution

The best way to analyze this problem is by asking the question, “What is the next palindrome bigger than 2002?” Since the required palindrome should be of the form $2aa2$, where the middle two digits (both a) do not equal 0, it must be the number 2112. Thus, the number that must be added to 2002 is $2112 - 2002 = 110$. ANSWER: (B)

14. The first six letters of the alphabet are assigned values A = 1, B = 2, C = 3, D = 4, E = 5, and F = 6. The value of a word equals the sum of the values of its letters. For example, the value of BEEF is 2 + 5 + 5 + 6 = 18. Which of the following words has the greatest value?

(**A**) BEEF (**B**) FADE (**C**) FEED (**D**) FACE (**E**) DEAF

Solution

Each of the five given words contains both an “E” and an “F”, so we can eliminate these letters for the purposes of making the comparison. So after eliminating these letters we are looking at the 5 “words”,

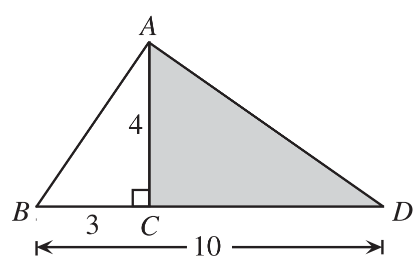
(A) BE **(B)** AD **(C)** ED **(D)** AC **(E)** DA

The highest value of these five words is ED, which has a value of 9, thus implying that FEED has the highest value of the original five words.

Alternatively, we could have calculated the value of each of the five words, and again seen that FEED has the highest value. ANSWER: (C)

15. In the diagram, $AC = 4$, $BC = 3$, and $BD = 10$. The area of the shaded triangle is

(**A**) 14 (**B**) 20 (**C**) 28 (**D**) 25 (**E**) 12



Solutionn

If $BD = 10$ and $BC = 3$, then $CD = 7$. The area of the shaded triangle is $\frac{1}{2}(7)(4) = 14$.

ANSWER: (A)

16. In the following equations, the letters a, b and c represent different numbers.

$$1^3 = 1$$
$$a^3 = 1 + 7$$
$$3^3 = 1 + 7 + b$$
$$4^3 = 1 + 7 + c$$

The numerical value of $a + b + c$ is

(**A**) 58 (**B**) 110 (**C**) 75 (**D**) 77 (**E**) 79

Solution

Since $2^3 = 8 = 1 + 7$, then $a = 2$.

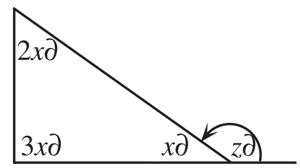
Since $3^3 = 27$, then $27 = 8 + b$ or $b = 19$.

Since $4^3 = 64$, then $64 = 8 + c$ or $c = 56$.

Thus, $a + b + c = 2 + 19 + 56 = 77$. ANSWER: (D)

17. In the diagram, the value of z is

(**A**) 150 (**B**) 180 (**C**) 60
(**D**) 90 (**E**) 120



Solution

Since the angles of a triangle add to 180∂,

$$2x\partial + 3x\partial + x\partial = 180\partial$$
$$6x\partial = 180\partial$$
$$x = 30$$

Now since the angles $x\partial$ and $z\partial$ together form a straight line, the

$$x\partial + z\partial = 180\partial$$
$$30\partial + z\partial = 180\partial$$
$$z = 150$$

ANSWER: (A)

18. A perfect number is an integer that is equal to the sum of all of its positive divisors, except itself. For example, 28 is a perfect number because $28 = 1 + 2 + 4 + 7 + 14$. Which of the following is a perfect number?

(**A**) 10 (**B**) 13 (**C**) 6 (**D**) 8 (**E**) 9

Solution

We must check each of the answers:

	Number	Positive divisors	Sum of all positive divisors except itself
(A)	10	1, 2, 5, 10	$1 + 2 + 5 = 8$
(B)	13	1, 13	1
(C)	6	1, 2, 3, 6	$1 + 2 + 3 = 6$
(D)	8	1, 2, 4, 8	$1 + 2 + 4 = 7$
(E)	9	1, 3, 9	$1 + 3 = 4$

The only number from this set that is a perfect number is 6. (Note that the next two perfect number bigger than 28 are 496 and 8128.) ANSWER: (C)

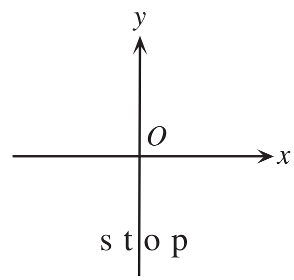
19. Subesha wrote down Davina's phone number in her math binder. Later that day, while correcting her homework, Subesha accidentally erased the last two digits of the phone number, leaving 893-44_ _. Subesha tries to call Davina by dialing phone numbers starting with 893-44. What is the least number of phone calls that she has to make to be guaranteed to reach Davina's house?

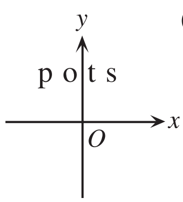
(**A**) 100 (**B**) 90 (**C**) 10 (**D**) 1000 (**E**) 20

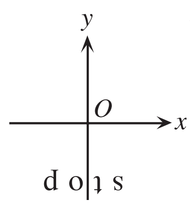
Solution

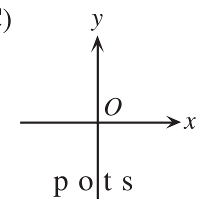
Davina could have a telephone number between and including 893-4400 and 893-4499. Since there are 100 numbers between and including these two numbers, this is precisely the number of calls that Subesha would have to make to be assured that she would reach Davina's house. An alternate way of seeing this is to realize that Davina's number is of the form 8 9 3 - 4 4 $\underline{a}$ $\underline{b}$, where there are 10 possibilities for a and for each of these possibilities, there are 10 possibilities for b. Thus there are 10 ¥ 10 = 100 different possibilities in total. Answer: (A)

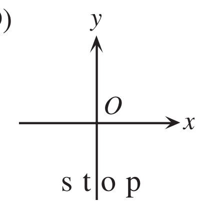
20. The word "stop" starts in the position shown in the diagram to the right. It is then rotated 180∂ clockwise about the origin, O, and this result is then reflected in the x-axis. Which of the following represents the final image?

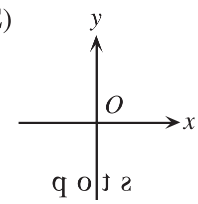


(**A**)


(**B**)


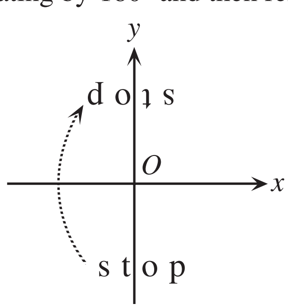
(**C**)


(**D**)


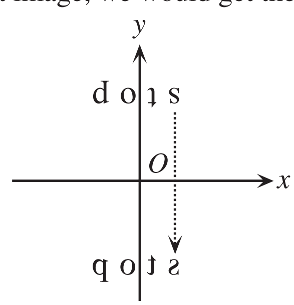
(**E**)


Solution

If we start by rotating by 180° and then reflecting that image, we would get the following:



Rotation of 180°



Reflection in x-axis

Answer: (E)

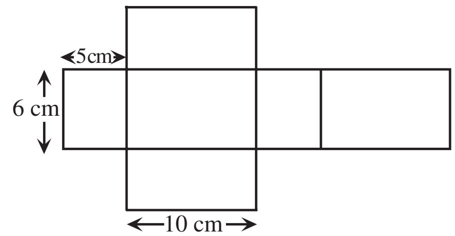
Part C

21. Five people are in a room for a meeting. When the meeting ends, each person shakes hands with each of the other people in the room exactly once. The total number of handshakes that occurs is
(**A**) 5 (**B**) 10 (**C**) 12 (**D**) 15 (**E**) 25

Solution

Each of the five people in the room will shake four others' hands. This gives us 20 handshakes, except each handshake is counted twice (Person X shakes Person Y's hand *and* Person Y shakes Person X's hand), so we have to divide the total by 2, to obtain 10 handshakes in total. Answer: (B)

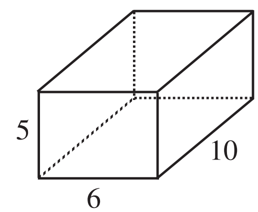
22. The figure shown can be folded along the lines to form a rectangular prism. The surface area of the rectangular prism, in cm^2, is



(**A**) 312 (**B**) 300 (**C**) 280
(**D**) 84 (**E**) 600

Solution

The required surface area is $2(5 ¥ 6 + 5 ¥ 10 + 6 ¥ 10) = 280\text{ cm}^2$.
Alternatively, if we fold the net into a rectangular box, we would obtain the following diagram. From this we can see that the faces of the box are two rectangles of area 30 cm^2, two rectangles of area 50 cm^2, and two rectangles of area 60 cm^2. This gives a total surface area of 280 cm^2.



Answer: (C)

23. Mark has a bag that contains 3 black marbles, 6 gold marbles, 2 purple marbles, and 6 red marbles. Mark adds a number of white marbles to the bag and tells Susan if she now draws a marble at random from the bag, the probability of it being black or gold is $\frac{3}{7}$. The number of white marbles that Mark adds to the bag is

(**A**) 5 (**B**) 2 (**C**) 6 (**D**) 4 (**E**) 3

Solution

Since the probability of selecting a black or gold marble is $\frac{3}{7}$, this implies that the total number of marbles in the bag is a multiple of 7. That is to say, there are possibly 7, 14, 21, 28, etc. marbles in the bag. The only acceptable number of marbles in the bag is 21, since there are 9 marbles in total which are black or gold, and $\frac{9}{21} = \frac{3}{7}$. If there are 21 marbles in the bag, this means that 4 marbles must have been added, since there are 17 already accounted for.
Alternatively, we could say that the number of white marbles in the bag was w (an unknown number), and form the equation

$$\frac{6+3}{17+w} = \frac{3}{7}$$

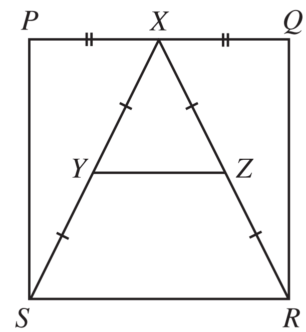
$$\frac{9}{17+w} = \frac{3}{7}$$

$$\frac{9}{17+w} = \frac{9}{21}, \text{ changing to a numerator of 9.}$$

Thus, $17 + w = 21$ or $w = 4$, and so the number of white marbles is 4. Answer: (D)

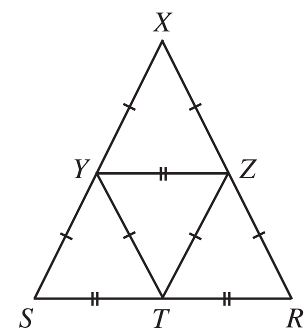
24. *PQRS* is a square with side length 8. *X* is the midpoint of side PQ, and *Y* and *Z* are the midpoints of *XS* and *XR*, respectively, as shown. The area of trapezoid *YZRS* is

(**A**) 24 (**B**) 16 (**C**) 20
(**D**) 28 (**E**) 32



Solution

If *PQRS* is a square with side length 8, it must have an area of 64 square units. The area of $\triangle XSR$ is thus $\frac{1}{2}(8)(8) = 32$. If we take the point *T* to be the midpoint of *SR* and join *Y* and *Z* to *T*, we would have the following diagram.



Each of the four smaller triangles contained within $\triangle XSR$ has an equal area, which is therefore $\frac{1}{4}(32) = 8$. Since the area of trapezoid *YZRS* is made up of three of these triangles, it has an area of $3 \times 8 = 24$.

ANSWER: (A)

25. Each of the integers 226 and 318 have digits whose product is 24. How many three-digit positive integers have digits whose product is 24?

(**A**) 4 (**B**) 18 (**C**) 24 (**D**) 12 (**E**) 21

Solution

First, we determine all of the possible ways to write 24 as the product of single-digit numbers.

(i) $24 = 1 \times 4 \times 6$
(ii) $24 = 1 \times 3 \times 8$
(iii) $24 = 2 \times 3 \times 4$
(iv) $24 = 2 \times 2 \times 6$

The cases numbered (i), (ii) and (iii) each give 6 possible arrangements. For example, if we consider $24 = 1 \times 4 \times 6$, the 6 possibilities are then 146, 164, 416, 461, 614, and 641. So for cases (i), (ii) and (iii), we have a total of 18 possibilities.

For the fourth case, there are only 3 possibilities, which are 226, 262 and 622.

In total there are $18 + 3 = 21$ possibilities. ANSWER: (E)

* * * * *